D3-brane from D-instantons in the hybrid formalism of superstring

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Abstract

A Dp-brane can be regarded as a configuration of infinitely many D$(p-2k)$-branes in bosonic string. We will show this property of D-branes in the superstring case using the hybrid formalism. It is convenient to study the boundary state for such D-branes to study such property between D-branes. We show that the boundary state for a D3-brane in a constant self-dual gauge field background can be expressed in terms of the boundary state for D-instantons in the hybrid formalism.

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§1. Introduction

Field theories on the noncommutative spacetime have been studied extensively. In the bosonic string theory, the noncommutativity of the bosonic coordinates arises at the end points of the open string on a D-brane in a constant NS-NS two form field background [1),2)]. This noncommutativity can be understood from a different point of view. A Dp-brane with a constant gauge field background can be regarded as a configuration of infinitely many D$(p-2k)$-branes [3),4)]. From this point of view, the noncommutativity is expressed as the noncommutativity of the matrix coordinates of D$(p-2k)$-branes. It is convenient to study the boundary state for such D-branes to study such property between D-branes. Indeed, we can see from the boundary state that the Seiberg-Witten map [5)] between commutative and noncommutative gauge fields can be regarded as the map between two different pictures of the D-brane [6)]. Furthermore, this point of view supports matrix models which treat lower dimensional D-branes as the fundamental degrees of freedom.

Recently, field theories on the deformed superspace have been considered, in which fermionic coordinates are non(anti)commutative [7),8)]. In the superstring theory, this nonanticommutativity arises in a constant self-dual graviphoton background [7),8),9),10)]. The graviphoton field corresponds to a RR vertex operator. Therefore, the hybrid formalism [11)] instead of the NSR formalism is useful to study such a background. The non(anti)commutativity of the fermionic coordinates arise at the end points of the open string.

We expect that the fermionic non(anti)commutativity can also be understood by using lower dimensional D-branes. If we can do so, we may be able to discuss the Seiberg-Witten map [12)] and supermatrix models [13),14),15),16)]. As a first step, we consider whether the boundary state for a Dp-brane in a constant gauge field background can be represented by that of a configuration of infinitely many D-instantons, using the hybrid variables.

This paper is organized as follows. In section 2, we review how to express the relation between a Dp-brane and infinitely many D$(p-2k)$-branes in the bosonic string theory. In section 3, we show that the boundary state for a D3-brane in a constant self-dual gauge field background can be expressed in terms of the boundary state for D-instantons in the hybrid superstring. Section 4 is devoted to conclusions and discussions.

§2. Dp-brane from D$(p-2k)$-branes

In this section, we review how a Dp-brane can be expressed as a configuration of infinitely many D$(p-2k)$-branes. In order to show this fact, the easiest way is to study the boundary state. For simplicity, we treat the case of $p=1, k=1$. It is straightforward to generalize to

other cases.

Let us consider the boundary state for a D1-brane in the bosonic string. A configuration of infinitely many D-instantons is represented by $\infty \times \infty$ matrices X^m. We will consider the configuration of D-instantons such as

$$\begin{aligned} X^1 &= \hat{M}^1, \\ X^2 &= \hat{M}^2, \end{aligned} \tag{1}$$

where the matrices $\hat{M}^1, \hat{M}^2$ satisfy

$$[\hat{M}^1, \hat{M}^2] = i\theta. \tag{2}$$

Here θ is a constant. The boundary state corresponding to this configuration is expressed as

$$|B\rangle = TrP\exp(-i\int_0^{2\pi} p^m(\sigma)\hat{M}_m)|B\rangle_{-1}, \tag{3}$$

where P denotes the path ordering with respect to σ and p^m is the conjugate momentum to x^m. $|B\rangle_{-1}$ denotes the boundary state for a D-instanton at the origin, $x^m(\sigma)|B\rangle_{-1} = 0$. This boundary state can be rewritten as a path integral representation,

$$|B\rangle = \int [dY^1 dY^2] \exp[\frac{i}{\theta}\int d\sigma Y^2 \partial_\sigma Y^1 - i\int d\sigma(p^1 Y^1 + p^2 Y^2)]|B\rangle_{-1}. \tag{4}$$

It is easy to see that this boundary state coincides with the boundary state for a D1-brane with a constant gauge field background $F_{21} = \frac{1}{\theta}$. Indeed, the following identities hold:

$$\begin{aligned} 0 &= \int [dY^1 dY^2] \frac{\delta}{\delta Y^1} \exp[\frac{i}{\theta}\int d\sigma Y^2 \partial_\sigma Y^1 - i\int d\sigma(p^1 Y^1 + p^2 Y^2)]|B\rangle_{-1} \\ &= [-\frac{i}{\theta}\partial_\sigma x^2 - ip^1]|B\rangle, \\ 0 &= \int [dY^1 dY^2] \frac{\delta}{\delta Y^2} \exp[\frac{i}{\theta}\int d\sigma Y^1 \partial_\sigma Y^2 - i\int d\sigma(p^1 Y^1 + p^2 Y^2)]|B\rangle_{-1} \\ &= [\frac{i}{\theta}\partial_\sigma x^1 - ip^2]|B\rangle. \end{aligned} \tag{5}$$

Therefore, this boundary state coincides with

$$\exp[\frac{i}{\theta}\int d\sigma(x^1 \partial_\sigma x^2)]|B\rangle_1, \tag{6}$$

up to normalization, where $|B\rangle_1$ denotes the boundary state for a D1-brane satisfying $p_1|B\rangle_1 = p_2|B\rangle_1 = x_3|B\rangle_1 = \cdots = x_D|B\rangle_1 = 0$.

§3. D3-brane from D-instantons in the hybrid formalism

In this section, we show that the boundary state for a D3-brane in a constant self-dual gauge field background can be expressed in terms of the boundary state for infinitely many D-instantons in the hybrid formalism.

Here, we use the hybrid superstring on $R^4 \times X^6$ [11]. The world-sheet action*) in a constant self-dual gauge field background is given by

$$S = \frac{1}{\pi}\int d\tau d\sigma\big(\frac{1}{2}\partial y^m \tilde{\partial} y_m - q_\alpha \tilde{\partial}\theta^\alpha + \bar{d}_{\dot{\alpha}}\tilde{\partial}\bar{\theta}^{\dot{\alpha}} - \frac{1}{2}\partial\rho\tilde{\partial}\rho - \tilde{q}_\alpha \partial\tilde{\theta}^\alpha + \tilde{\bar{d}}_{\dot{\alpha}}\partial\tilde{\bar{\theta}}^{\dot{\alpha}} - \frac{1}{2}\partial\tilde{\rho}\tilde{\partial}\tilde{\rho}\big) + S_C + S_F, \quad (7)$$

where S_C is the action for the variables corresponding to X^6. S_F represents a constant self-dual gauge field background, which is given by

$$S_F = -\frac{i}{2}\int d\sigma F_{mn}\Big\{y^m \partial_\sigma y^n + iq^\alpha \sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}^{n\dot{\alpha}\beta}\theta_\beta - i\tilde{q}^\alpha \sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}^{n\dot{\alpha}\beta}\tilde{\theta}_\beta\Big\}. \quad (8)$$

Before we go on, we need to mention the equations satisfied by the boundary state for a D3-brane. In a flat background, the boundary state $|B\rangle_{3f}$ satisfies the following equations,

$$\begin{aligned} (\partial + \tilde{\partial})y^m |B\rangle_{3f} &= 0, \\ (q^\alpha + \tilde{q}^\alpha)|B\rangle_{3f} = (\theta^\alpha - \tilde{\theta}^\alpha)|B\rangle_{3f} &= 0, \\ (\bar{d}^{\dot{\alpha}} + \tilde{\bar{d}}^{\dot{\alpha}})|B\rangle_{3f} = (\bar{\theta}^{\dot{\alpha}} - \tilde{\bar{\theta}}^{\dot{\alpha}})|B\rangle_{3f} &= 0, \\ (e^\rho + e^{\tilde{\rho}})|B\rangle_{3f} &= 0. \end{aligned} \quad (9)$$

In a constant self-dual gauge field background, the equations for the bosonic fields are modified as

$$\{\frac{1}{4\pi}(\partial + \tilde{\partial})y^m + F_{mn}(\partial - \tilde{\partial})y^n\}|B\rangle_3 = 0. \quad (10)$$

We can obtain the equations for the fermionic fields from the equations for the NSR variables because the map between the hybrid variables and the NSR variables is known. Here instead, we utilize the N=2 world-sheet supersymmetry to determine the equations for the fermionic fields. The N=2 supersymmetry algebra consists of the generators,

$$\begin{aligned} T &= \frac{1}{2}\partial y^m \partial y^m - q_\alpha \partial\theta^\alpha + \bar{d}_{\dot{\alpha}}\partial\bar{\theta}^{\dot{\alpha}} - \frac{1}{2}\partial\rho\partial\rho + \frac{1}{2}\partial^2\rho + T_C + \frac{i}{2}\partial J_C, \\ G^+ &= e^\rho d_\alpha d^\alpha + G^+_C, \\ G^- &= e^{-\rho}\bar{d}_{\dot{\alpha}}\bar{d}^{\dot{\alpha}} + G^-_C, \\ J &= i\partial\rho + J_C, \end{aligned} \quad (11)$$

*) We use the convention of ref.[8]

where

$$d_\alpha = q_\alpha + 2i\sigma^n_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial y_n - 4\bar\theta\bar\theta\partial\theta_\alpha, \tag{12}$$

and $[T_C, G^+_C, G^-_C, J_C]$ are the c=9 N=2 superconformal generators of the compactified space X^6. We determine the equations for the fermionic fields using the fact that the boundary state should preserve this symmetry. Since the boundary action in eq.(8) does not involve ρ, the boundary state $|B\rangle_3$ satisfies

$$(e^\rho + e^{\tilde\rho})|B\rangle_3 = 0, \tag{13}$$

as in the flat case. The world-sheet variables transform under the symmetry corresponding to $\int(G^+ + \tilde G^+)$ as

$$\begin{aligned}
&\delta_G\theta^\alpha = e^\rho d^\alpha, \qquad \delta_G\tilde\theta^\alpha = e^{\tilde\rho}\tilde d^\alpha,\\
&\delta_G y^m = 2ie^\rho d^\alpha\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} + 2ie^{\tilde\rho}\tilde d^\alpha\sigma^m_{\alpha\dot\alpha}\tilde{\bar\theta}^{\dot\alpha},\\
&\delta_G q^\alpha = -4\partial(e^\rho d^\alpha\bar\theta\bar\theta), \qquad \delta_G\tilde q^\alpha = -4\tilde\partial(e^{\tilde\rho}\tilde d^\alpha\tilde{\bar\theta}\tilde{\bar\theta}).
\end{aligned} \tag{14}$$

From (10),(13) and (14), we determine the equations for the fermionic fields as

$$\begin{aligned}
&\{F_{nm}\sigma^m_{\alpha\dot\alpha}\bar\sigma^{n\dot\alpha\beta}(q^\alpha - \tilde{q^\alpha}) - \frac{1}{\pi}(q^\beta + \tilde{q^\beta})\}|B\rangle_3 = 0,\\
&\{F_{nm}\sigma^m_{\alpha\dot\alpha}\bar\sigma^{n\dot\alpha\beta}(\theta^\alpha + \tilde\theta^\alpha) - \frac{1}{\pi}(\theta^\beta - \tilde\theta^\beta)\}|B\rangle_3 = 0,\\
&(\bar d^{\dot\alpha} + \tilde{\bar d}^{\dot\alpha})|B\rangle_3 = (\bar\theta^{\dot\alpha} - \tilde{\bar\theta}^{\dot\alpha})|B\rangle_3 = 0.
\end{aligned} \tag{15}$$

3.1. *Boundary state for infinitely many D-instantons*

We consider the following boundary state as a generalization of eq.(3),

$$|B\rangle = TrP\exp[-\frac{1}{4\pi}\int d\sigma\{(\partial+\tilde\partial)y^m\hat M_m + (\partial\theta_\alpha + \tilde\partial\tilde\theta_\alpha)\hat Q^\alpha + (q^\alpha + \tilde{q^\alpha})\hat\Theta_\alpha\}]|B\rangle_{-1}, \tag{16}$$

where $\hat M_m, \hat Q^\alpha, \hat\Theta_\alpha$ are $\infty\times\infty$ matrices. $|B\rangle_{-1}$ denotes the boundary state for a D-instanton satisfying

$$\begin{aligned}
&y^m|B\rangle_{-1} = 0,\\
&(q^\alpha - \tilde q^\alpha)|B\rangle_{-1} = (\theta^\alpha + \tilde\theta^\alpha)|B\rangle_{-1} = 0,\\
&(\bar d^{\dot\alpha} + \tilde{\bar d}^{\dot\alpha})|B\rangle_{-1} = (\bar\theta^{\dot\alpha} - \tilde{\bar\theta}^{\dot\alpha})|B\rangle_{-1} = 0.
\end{aligned} \tag{17}$$

As in the bosonic case, $\int d\sigma(\partial+\tilde\partial)y^m$, $\int d\sigma(q^\alpha + \tilde{q^\alpha})$ and $\int d\sigma(\partial\theta_\alpha + \tilde\partial\tilde\theta_\alpha)$ should be identified as the vertex operators for a D-instanton. In the hybrid formalism, an integrated vertex operator is given by

$$\int dz G^-(G^+(V)). \tag{18}$$

Here V is a function of $\theta, \bar{\theta}$ for the vertex operator in the D-instanton case. The vertex operators $\int d\sigma(\partial+\tilde{\partial})y^m$, $\int d\sigma(q^\alpha+\tilde{q^\alpha})$, $\int d\sigma(\partial\theta_\alpha+\tilde{\partial}\tilde{\theta}_\alpha)$ can be obtained from $V=\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$, $\theta^\alpha\bar{\theta}\bar{\theta}$, θ^α respectively. $\int d\sigma(\partial+\tilde{\partial})y^m$ and $\int d\sigma(q^\alpha+\tilde{q^\alpha})$ generate the translation of y^m and $(\theta^\alpha+\tilde{\theta}^\alpha)$ respectively and the vertex operator $\int d\sigma(\partial\theta_\alpha+\tilde{\partial}\tilde{\theta}_\alpha)$ is a BRST exact operator generating the translation of a gauge degree of freedom. Therefore, $\hat{M}_m, \hat{Q}^\alpha, \hat{\Theta}_\alpha$ represent a configuration of infinitely many D-instantons and (16) is the boundary state corresponding to such a configuration. Here we consider the configuration in which $\hat{M}_m, \hat{Q}^\alpha, \hat{\Theta}_\alpha$ satisfy

$$
\begin{aligned}
[\hat{M}^n, \hat{M}^m] &= i\theta^{nm}, \\
\{\hat{\Theta}_\alpha, \hat{Q}^\beta\} &= \frac{i}{4}\sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}^{n\dot{\alpha}\beta}\theta^{nm},
\end{aligned}
\tag{19}
$$

where θ^{nm} is self-dual. The boundary state can be rewritten as the following path integral representation,

$$
\begin{aligned}
|B\rangle = \int[dY d\chi d\vartheta]\prod_{\alpha,\gamma}\chi_0^\gamma\{F_{mn}\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\vartheta_{\beta 0}+\frac{1}{\pi}(\theta_{\alpha 0}-\tilde{\theta}_{\alpha 0})\} \\
\times\exp[\frac{i}{2}\int d\sigma F_{mn}\{Y^m\partial_\sigma Y^n-\frac{1}{2}\chi^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\partial_\sigma\vartheta_\beta\} \\
-\frac{1}{4\pi}\int d\sigma\{(\partial+\tilde{\partial})y^m Y_m+(\partial\theta_\alpha+\tilde{\partial}\tilde{\theta}_\alpha)\chi^\alpha+(q^\alpha+\tilde{q^\alpha})\vartheta_\alpha\}]|B\rangle_{-1},
\end{aligned}
\tag{20}
$$

where $F_{mn}=(\theta^{nm})^{-1}$. ϑ, χ are periodic with respect to σ and $\theta_{\alpha 0}, \vartheta_{\alpha 0}, \chi_{\alpha 0}$ are the zero modes of $\theta_\alpha=\sum_n\theta_{\alpha n}e^{in\sigma}, \vartheta_\alpha=\sum_n\vartheta_{\alpha n}e^{in\sigma}, \chi_\alpha=\sum_n\chi_{\alpha n}e^{in\sigma}$ respectively. The eq.(20) is the generalization of eq.(4) except for the zero mode insertions of the fermionic fields. In our case, we need such insertions to express the Tr of the Chan-Paton factors in terms of the path integral representation. Since the zero mode does not appear in the exponent of eq.(20), we can understand the necessity of $\chi_{\alpha 0}$ insertion. Although it is not clear why $\vartheta_{\alpha 0}$ insertion is required, it becomes clear from the open string picture.

In this picture, the action in the background (19) can be guessed from (20) as

$$
\begin{aligned}
S = S_0+\frac{i}{2}\int d\tau F_{mn}\{Y^m\partial_\tau Y^n-\frac{1}{2}\chi^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\partial_\tau\vartheta_\beta\} \\
+\frac{i}{4\pi}\int d\tau\{(\partial-\tilde{\partial})y^m Y_m+(\partial\theta_\alpha-\tilde{\partial}\tilde{\theta}_\alpha)\chi^\alpha+(q^\alpha-\tilde{q^\alpha})\vartheta_\alpha\},
\end{aligned}
\tag{21}
$$

where S_0 is the action of the free fields and $Y^m, \chi^\alpha, \vartheta_\beta$ are the Chan-Paton degrees of freedom on the boundaries. Let us consider the case, $F_{mn}\to\infty$. Redefining the fields appropriately, the boundary action is given by

$$
\int d\tau(Y^1\partial_\tau Y^2+Y^3\partial_\tau Y^4+\chi^\alpha\partial_\tau\vartheta_\alpha). \tag{22}
$$

Canonically quantizing them, the Hilbert space is of the form $\mathcal{H}_Y\otimes\mathcal{H}_\vartheta$. The basis of $\mathcal{H}_Y$ can be taken to be the eigenstates of Y^1, Y^3 and the basis of $\mathcal{H}_\vartheta$ can be taken to

be $|0\rangle, \vartheta_\alpha|0\rangle, \vartheta^\alpha\vartheta_\alpha|0\rangle$ where $|0\rangle$ is defined by $\chi_\alpha|0\rangle = 0$. Since (22) corresponds to the background (19) with $\theta^{mn} = 0$ and represent infinitely many independent D-instantons, we expect that the boundary state should be the one for one D-instanton multiplied by ∞. By the following discussion, we can show that the insertion of ϑ_α is required in order to reproduce such result. The simple trace over $\mathcal{H}_\vartheta$ is

$$Tr_s\mathcal{O} = \frac{1}{2}\Big\langle 0|\mathcal{O}\vartheta^\alpha\vartheta_\alpha|0\Big\rangle + \Big\langle 0|\vartheta^\alpha\mathcal{O}\vartheta_\alpha|0\Big\rangle + \frac{1}{2}\Big\langle 0|\vartheta^\alpha\vartheta_\alpha\mathcal{O}|0\Big\rangle. \tag{23}$$

Since ϑ, χ are anticommuting fields, this trace corresponds to the following path integral,

$$\int [d\vartheta d\chi]_{a.p}\mathcal{O}e^{\int d\tau \chi^\alpha\partial_\tau\vartheta_\alpha}, \tag{24}$$

where ϑ, χ are antiperiodic with respect to τ. In order to obtain the periodic boundary condition, the trace should be weighted by $(-1)^F$ where F is the fermion number, i.e.,

$$\begin{aligned} Tr_s(-1)^F\mathcal{O} &= \frac{1}{2}\Big\langle 0|(-1)^F\mathcal{O}\vartheta^\alpha\vartheta_\alpha|0\Big\rangle + \Big\langle 0|\vartheta^\alpha(-1)^F\mathcal{O}\vartheta_\alpha|0\Big\rangle + \frac{1}{2}\Big\langle 0|\vartheta^\alpha\vartheta_\alpha(-1)^F\mathcal{O}|0\Big\rangle \\ &= \int [d\vartheta d\chi]_p\mathcal{O}e^{\int d\tau\chi^\alpha\partial_\tau\vartheta_\alpha}. \end{aligned} \tag{25}$$

Here ϑ, χ are now periodic with respect to τ. When $\mathcal{O}$ is the identity operator 1, this trace is zero. In order to make it non zero, we insert $\chi^\alpha\chi_\alpha\vartheta^\beta\vartheta_\beta$ because

$$\begin{aligned} Tr_s\chi^\alpha\chi_\alpha\vartheta^\beta\vartheta_\beta(-1)^F 1 &= \frac{1}{2}\Big\langle 0|\chi^\alpha\chi_\alpha\vartheta^\beta\vartheta_\beta\vartheta^\gamma\vartheta_\gamma|0\Big\rangle \\ &\qquad - \Big\langle 0|\vartheta^\gamma\chi^\alpha\chi_\alpha\vartheta^\beta\vartheta_\beta\vartheta_\gamma|0\Big\rangle + \frac{1}{2}\Big\langle 0|\vartheta^\gamma\vartheta_\gamma\chi^\alpha\chi_\alpha\vartheta^\beta\vartheta_\beta|0\Big\rangle \\ &= \frac{1}{2}\Big\langle 0|\vartheta^\gamma\vartheta_\gamma\chi^\alpha\chi_\alpha\vartheta^\beta\vartheta_\beta|0\Big\rangle \neq 0. \end{aligned} \tag{26}$$

Since the trace over $\mathcal{H}_Y$ can be found to be infinite, we can obtain the desire result. In the closed string picture, this trace leads to the following boundary state,

$$\int [dYd\chi d\vartheta]\prod_{\alpha,\gamma}\chi_0^\gamma\vartheta_0^\alpha e^{\int d\sigma(Y^1\partial_\sigma Y^2 + Y^3\partial_\sigma Y^4 + \chi^\alpha\partial_\sigma\vartheta_\alpha)}|B\Big\rangle_{-1}, \tag{27}$$

which coincides with the boundary state for infinitely many independent D-instantons. For the general background where the action is given by (21), the insertion is modified as χp_χ where p_χ is the canonical conjugate of χ, i.e. $p_{\chi^\alpha} = \frac{1}{\pi}(\theta_\alpha - \tilde{\theta}_\alpha) + F_{mn}\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\vartheta_\beta$.

We would like to show that this boundary state corresponds to the boundary state for a D3-brane in a constant self-dual gauge field F_{mn} background. Indeed, $|B\rangle$ satisfies the following equations for the non zero mode,

$$0 = \int [dYd\chi d\vartheta]\frac{\delta}{\delta Y^l}\exp[\frac{i}{2}\int d\sigma F_{mn}\{Y^m\partial_\sigma Y^n - \frac{1}{2}\chi^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\partial_\sigma\vartheta_\beta\} \tag{28}$$

$$
\begin{aligned}
&\qquad -\frac{1}{4\pi}\int d\sigma\{(\partial+\tilde{\partial})y^m Y_m+(\partial\theta^\alpha+\tilde{\partial}\tilde{\theta}^\alpha)\chi_\alpha+(q^\alpha+\tilde{q^\alpha})\vartheta_\alpha\}]|B\Big\rangle_{-1}\\
&=\{\frac{1}{4\pi}(\partial+\tilde{\partial})y^l+F_{ln}(\partial-\tilde{\partial})y^n\}|B\Big\rangle,\\
0&=\int[dYd\chi d\vartheta]\frac{\delta}{\delta\vartheta_\gamma}\exp[\frac{i}{2}\int d\sigma F_{mn}\{Y^m\partial_\sigma Y^n-\frac{1}{2}\chi^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\partial_\sigma\vartheta_\beta\}\\
&\qquad -\frac{1}{4\pi}\int d\sigma\{(\partial+\tilde{\partial})y^m Y_m+(\partial\theta^\alpha+\tilde{\partial}\tilde{\theta}^\alpha)\chi_\alpha+(q^\alpha+\tilde{q^\alpha})\vartheta_\alpha\}]|B\Big\rangle_{-1}\\
&=\{F_{mn}\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\gamma}(q^\alpha-\tilde{q^\alpha})-\frac{1}{\pi}(q^\gamma+\tilde{q^\gamma})\}|B\Big\rangle,\\
0&=\int[dYd\chi d\vartheta]\frac{\delta}{\delta\chi^\gamma}\exp[\frac{i}{2}\int d\sigma F_{mn}\{Y^m\partial_\sigma Y^n-\frac{1}{2}\chi^\alpha\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\partial_\sigma\vartheta_\beta\}\\
&\qquad -\frac{1}{4\pi}\int d\sigma\{(\partial+\tilde{\partial})y^m Y_m+(\partial\theta^\alpha+\tilde{\partial}\tilde{\theta}^\alpha)\chi_\alpha+(q^\alpha+\tilde{q^\alpha})\vartheta_\alpha\}]|B\Big\rangle_{-1}\\
&=\{F_{mn}\sigma^n_{\gamma\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\alpha}(\partial\theta_\alpha-\tilde{\partial}\tilde{\theta}_\alpha)+\frac{1}{\pi}(\partial\theta_\gamma+\tilde{\partial}\tilde{\theta}_\gamma)\}|B\Big\rangle.
\end{aligned}
$$

Thus, we have shown $|B\rangle$ satisfies almost all in the equations (15) except for the ones involving $\theta_0, \tilde{\theta}_0$. Since (20) includes the following fact,

$$
\begin{aligned}
&\int[d\vartheta_0]\prod_\alpha\{F_{mn}\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}\vartheta_{\beta 0}+\frac{1}{\pi}(\theta_{\alpha 0}-\tilde{\theta}_{\alpha 0})\}e^{(q_0^\gamma+\tilde{q_0^\gamma})\vartheta_{\gamma 0}}|B\Big\rangle_{-1} \qquad (29)\\
&=\int[d\vartheta_0]\prod_\alpha\{F_{mn}\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}(\theta_{\beta 0}+\tilde{\theta}_{\beta 0})+\frac{1}{\pi}(\theta_{\alpha 0}-\tilde{\theta}_{\alpha 0})\}e^{(q_0^\gamma+\tilde{q_0^\gamma})\vartheta_{\gamma 0}}|B\Big\rangle_{-1}\\
&=\prod_{\alpha,\gamma}\{F_{mn}\sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}(\theta_{\beta 0}+\tilde{\theta}_{\beta 0})+\frac{1}{\pi}(\theta_{\alpha 0}-\tilde{\theta}_{\alpha 0})\}(q_0^\gamma+\tilde{q_0^\gamma})|B\Big\rangle_{-1},
\end{aligned}
$$

we can show that $|B\rangle$ satisfies the equations (15). Thus, the boundary state $|B\rangle$ coincides with for a D3-brane in a constant self-dual gauge field F_{mn} background.

§4. Conclusions and discussions

In this paper, we express the boundary state for a D3-brane in a constant self-dual gauge field background in terms of the boundary state for infinitely many D-instantons in the hybrid formalism. Therefore, a D3-brane in a constant self-dual gauge field background can be regarded as a configuration of infinitely many D-instantons in the hybrid formalism. In order to discuss the above statement more exactly, we need to study the open string theory corresponding to infinitely many D-instantons in the background (19). As mentioned above, we should pay attention to the zero mode insertions.

In the bosonic string, we can express the boundary state for a Dp-brane in a constant gauge field background in terms of the boundary state for D$(p-2k)$-branes. In the hybrid

formalism, we have discussed the case where $p = 3, k = 2$ and the constant gauge field is self-dual. Let us consider other cases where $p - 2k = -1$ and the constant gauge field is general. In such cases, the equations for the field $(\bar{d}^{\dot{\alpha}} + \tilde{\bar{d}}^{\dot{\alpha}})$ satisfied by the boundary state for a Dp-brane are different from those for D-instantons. In order to modify such boundary conditions, we should add $(\bar{q}^{\dot{\alpha}} - \tilde{\bar{q}}^{\dot{\alpha}})$ in (16). These operators are not free fields and do not commute with y^m and so on. Thus we can not obtain the equations as (28) easily. For this reason, it is difficult to decide the commutation relations of matrices as (19) to express the boundary state for a Dp-brane in terms of the boundary state for D-instantons. For a D3-brane in a constant self-dual gauge field background, since the equations for the field $(\bar{d}^{\dot{\alpha}} + \tilde{\bar{d}}^{\dot{\alpha}})$ are same as those for D-instantons, we can express the boundary state for a D3-brane in terms of the boundary state for D-instantons.

Our construction is the first step to understand the general non(anti)commutativity by lower dimensional D-branes. We would like to consider a D-brane in a graviphoton background using our construction in a separate publication.

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